Influence of Drag Reducing Additives on Mixing and Dispersing in Agitated Vessels

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The influence of the addition of drag reducing polymers and fibers on two transport processes in mechanically agitated liquid systems is examined. The first one concerns the determination of mixing times, which were found to be insignificantly to moderately increased in the drag reducing systems. The second one concerns the determination of the critical RPM at which complete suspension of solid particles occurs. It was shown that this was unaffected in the drag reducing systems. The results of this work can be applied to fermentation processes, biological reactors with flocs, and metallurgical processes. The theoretical implications of this work are to be seen in the association of polymeric and fibrous drag reducing additives with different parts of the turbulent spectrum.

SCOPE

Although a considerable body of knowledge exists on the reduction of turbulent skin friction in liquids by polymers, surfactants, and fibers, most of this work as reviewed recently by Hoyt (1972*a*, *b*), Virk (1975), and White (1976) refers either to internal flows in conduits or to external flows past regularly shaped bodies. Recently, a paper written by Quraishi et al. (1976) reports on the torque suppression by polymeric and fibrous drag reducing additives in mechanically stirred tanks. It is suggested that this phenomenon, apart from having theoretical implications, could be applied to those technologies where extremely small quantities of drag reducing additives are not harmful and where the presence of a second phase (gaseous or solid) actually enhances the suppression of torque.

The question of mass transfer and of the mixing performance was, however, in the work of Quraishi et al. (1976), left open. Since agitation is never done for its own sake, it must be highly desirable to find out what influence the drag reducing additives might exercise upon the transport processes in stirred tanks. Then and only then can the significance of using drag reducing additives in agitated tanks be properly assessed.

For aerobic wastewater treatment and for fermentation processes, the gas-liquid mass transfer of oxygen is of crucial importance. The work on these aspects is still in progress, and the results will be reported shortly. The scope of this paper is to report on the liquid-phase mixing and on the suspending of solid particles in stirred tanks with drag reducing additives.

CONCLUSIONS AND SIGNIFICANCE

Polyacrylamid and polyethylenoxide on their own and in combination with glass fibers were used to suppress the torque of standard six-blade turbines operating in two acrylic-glass tanks (0.24 and 0.30 m). The liquid-phase mixing was assessed by measuring mixing times, that is, the times required for an iodine-sodium thiosulfate reaction to reach completion. It was found that up to the concentration of 250 p.p.m. of a polymer, the increase of mixing time is insignificant. In 250 p.p.m. polyacrylamide solutions, the mixing times could be, however, increased by as much as a factor of 2. Still larger increases were found when glass fibers were used (2 500 p.p.m.), particularly in combination with polymers.

The interpretation of this phenomenon can follow along one of the three following lines: the increase of elongational viscosity, suppression of the trailing vortex system behind the blades, and the suppression of the microscale turbulence. It is shown that the conclusions drawn from these three models are not incompatible.

The fact that the large scale convective turbulence is not affected by either the polymeric or the fibrous additives was demonstrated by a series of experiments in which the critical impeller speed was measured necessary to suspend solid glass beads (1 and 2.5 mm). No difference was detected between the critical speeds in water, polymeric, and fibrous drag reducing additives.

Apart from the theoretical implications, the significance of this work is in the potential applications in certain biological reactors, in hydrometallurgy, and in flocculation. Drag reducing additives can be used in these applications without affecting the rate of suspension of solid particles. The fact that the small scale turbulence becomes reduced by the drag reducing additives can be beneficial in systems with shear sensitive flocs and microorganisms.

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In previous papers, Mashelkar et al. (1975) and Quraishi et al. (1976) investigated the influence of drag reducing additives (DRA) on the power consumption in agitated vessels. A variety of drag reducing additives (polymers, surfactants, and solid suspensions) were tested in aerated and unaerated vessels. The maximum torque suppression found in nonaerated tanks was about 59%, whereas in aerated tanks torque suppression by as much as 77% was observable (over and above the reduction of torque due to aeration alone). Quraishi et al. (1976) investigated the possibility of synergistic effects with the combination of polymers and fibers on the lines of the work of Lee et al. (1974) which was conducted in pipe flows. Striking differences in the synergism between the pipe flow and the agitated vessel systems were noted. The effect of the scale of the equipment was a particularly interesting one in that larger scales gave larger torque suppressions. This observation and the earlier reported significant torque suppression levels in agitated tanks implies that there is a strong possibility of exploiting these effects in industrial situations.

In industrial practice, agitated vessels will be invariably used for carrying out the processes of dispersion of gases, liquids (miscible and immiscible), and solids which may or may not be accompanied by mass and heat transfer with a chemical reaction. Although the reduction in power consumption can be certainly obtained by the addition of DRA, it is not clear as to whether or not the mass and heat transfer processes will be affected in any significant way. Indeed, since these transport processes depend crucially upon the hydrodynamics in the agitated vessel, and since the torque suppression is linked to an alteration in the hydrodynamics, it is natural to anticipate that such changes will probably occur. However, the directions in which such transport processes will be affected are not clear. It is precisely for this reason that the present study was undertaken.

The time for homogenization of two miscible liquids is an important quantity and it depends strongly on the hydrodynamics within the agitated vessel. This quantity is referred to as the mixing time. The definition of this quantity and the measurement techniques, models, and correlations have been described by Ford and Ulbrecht (1975) and will not be dealt with here in any great detail. It suffices to say that mixing time is defined by considering the time required for the nonhomogenities during scalar transport in an agitated vessel to reduce to an arbitrary level. Shorter mixing times are preferable since they lead to smaller overall power consumptions. We decided to study experimentally the mixing times in agitated vessels not only for their importance per se but also to see if such studies could throw some light on the mechanism of drag reduction in agitated vessels.

It is important to suspend solid particles in liquid media for a variety of reasons: the uniform distribution of insoluble solids in liquid media for the purpose of uniform withdrawal of slurry from the vessel (for example, sand in water), particle reduction, dissolution or leaching, solids formation, and reaction. In all these cases, the requirement of degree of homogenity may vary. For example, when a stirred vessel is used as a mixer from which the suspension must be fed to a chemical reactor, a high degree of homogenity may be required. In a batchwise dissolution of a solid in a liquid, however, a low homogenity may be adequate. In either case, however, it is important to ensure that all the solid particles are completely suspended in the liquid medium. When the solid particles are heavier than the liquid phase, they settle down to the bottom, and as the RPM is increased, more and more particles get entrained. At a critical RPM, however, all the particles get suspended. It is important to know this critical RPM for two main reasons. It has been shown experimentally that above this critical RPM, the solid-liquid mass transfer coefficient is quite insensitive to RPM. In the turbulent regime, the power consumption varies as RPM raised to the one third power. This would mean that a considerable energy would be wasted by operating at RPM's higher than the critical. Hence, we decided to study the influence of drag reducing additives on the critical RPM which is required to keep all the solids in suspension. As we shall show later, apart from its importance per se in certain operations (we discuss a hydrometallurgical operation in particular), we can also get some idea about the gross velocities in drag reducing fluids at the bottom of the agitated vessel.

Table 1. The Details of the Agitation Systems Used in This Work

Agitator	Diameter, m	Height of blade, m	Width of blade, m
Six-blade turbine	0.08	0.015	0.02
Six-blade turbine	0.10	0.020	0.025
Six-blade turbine	0.147	0.030	0.035
Paddle	0.152	0.028	—
Inclined blade paddle	0.150	0.018	0.035

EXPERIMENTAL

All the experiments were carried out in two cylindrical, flat bottom tanks made from glass and perspex and provided with four baffles. The diameters of these vessels were 24 and 30 cm, respectively. The agitators used were six-blade turbines, paddle, and inclined blade paddle. The geometrical details have been provided in Table 1.

The standard iodine-sodium thiosulphate method was used to measure the terminal mixing times. The salient experimental details are given by Ford and Ulbrecht (1975). The measurement is essentially comprised of recording the process of decoloration with a sensitive photocolorimeter and finding the time at which complete decolorization occurred. Approximately 260 data points were taken with a variety of systems. The experimental accuracy and reproductability of these measurements was about $\pm$ 10%.

The minimum stirrer speed at which the last particles were lifted from the bottom of the vessel was determined by a procedure described in detail by Zwietering (1958). In principle, the particle bed at the strongly illuminated tank bottom was viewed through an inclined mirror, and the RPM at which the last particle left the bottom was determined. The measurements were reproducible within $\pm$ 3%.

The polymer drag reducing additives were polyethylene oxide (PEO-WSR-301) and polyacrylamide (PAA-AP30) in concentrations of 50, 100, and 250 p.p.m. in water. The fibers used as drag reducing additives were glass fibers approximately 3 mm long and aspect ratio of 100. The glass fibers were treated with concentrated hydrochloride acid and then washed in water and rinsed with methanol. Two types of glass beads (1 and 2 mm diam.) were used at one concentration of 4.45% by weight. The density of the glass beads was 2.988 g/cm^3.

The rheological properties of the model systems were measured on a Weissenberg rheogoniometer (model R-18). All the experiments were conducted in a constant room temperature maintained at 22°C.

RESULTS AND DISCUSSION

Mixing Times in Drag Reducing Additive Solutions

The data obtained for mixing times with different solutions were converted to a dimensionless number F which is defined after Norwood and Metzner (1960):

$$F = \frac{N\theta}{(T/D)^{3/2}(H/D)^{1/2}(N^2D/g)^{1/6}} \qquad (1)$$

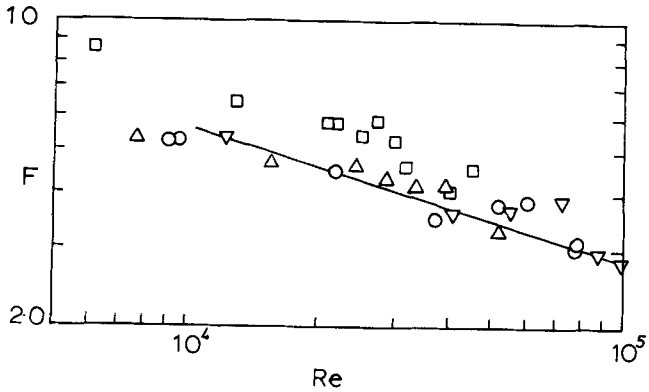

Fig. 1. Influence of DRA on dimensionless mixing times. Vessel diameter 0.30 m, stirrer diameter 0.10 m. ▽ water, ○ 50 p.p.m. PAA, △ 100 p.p.m. PAA, □ 250 p.p.m. PAA.

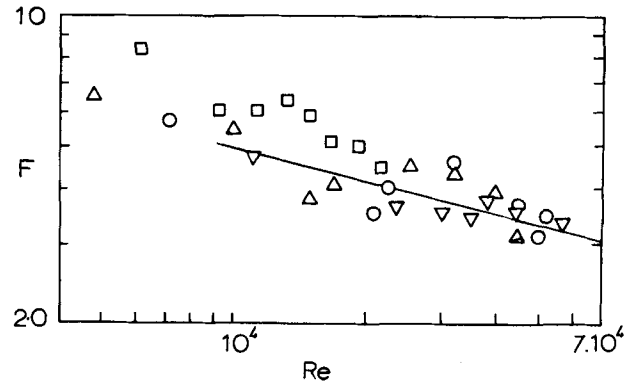

Fig. 2. Influence of DRA on dimensionless mixing times. Vessel diameter 0.24 m, stirrer diameter 0.08 m. ▽ water, ○ 50 p.p.m. PEO, △ 100 p.p.m. PEO, □ 250 p.p.m. PEO.

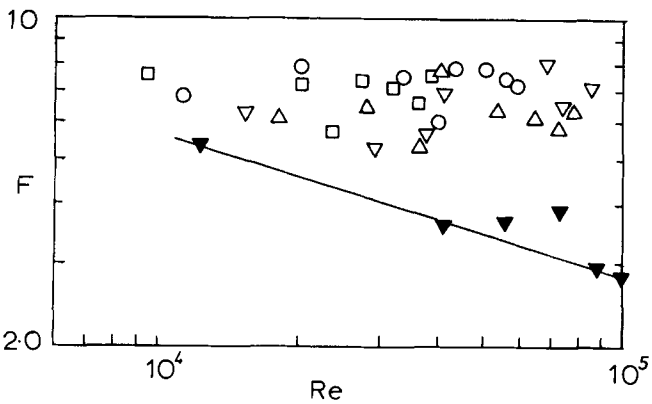

Fig. 3. Influence of DRA on dimensionless mixing times. Vessel diameter 0.30 m, stirrer diameter 0.10 m. ● water, ▽ water and fibers (2 500 p.p.m.), ○ 50 p.p.m. PAA and fibers (2 500 p.p.m.), △ 100 p.p.m. PAA and fibers (2 500 p.p.m.), □ 250 p.p.m. PAA and fibers (2 500 p.p.m.).

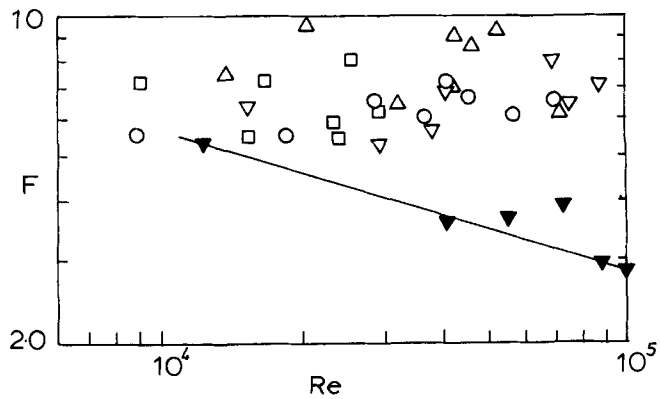

Fig. 4. Influence of DRA on dimensionless mixing times. Vessel diameter 0.30 m, stirrer diameter 0.10 m. ● water, ▽ water and fibers (2 500 p.p.m.), ○ 50 p.p.m. PEO and fibers (2 500 p.p.m.), △ 100 p.p.m. PEO and fibers (2 500 p.p.m.), □ 250 p.p.m. PEO and fibers (2 500 p.p.m.).

It was preferable to express the dimensionless mixing time in this form because the geometrical variation could be specifically taken into account. In Figures 1, 2, and 3, we have plotted F vs. Re. Since all the solutions and fiber suspensions used in this work showed a constant Newtonian viscosity, Re was defined in the usual way, and any influence of shear thinning viscosity may not be significant as far as the data interpretation is concerned.

The mixing time is obviously a very arbitrary parameter depending strongly upon the geometry of the system, upon the experimental technique used to monitor the transient response curve, and upon the selection of the end point. The method is useful for the sake of comparison only. Hence, we shall mainly investigate the influence of DRA on mixing times by using our own data on mixing times in water as the base. In spite of this, we compared the curve on mixing times obtained by Norwood and Metzner (1960). With our water data there was a good agreement between their curve and our water data, but in view of the foregoing comments, the agreement should be considered coincidental. Significantly, the trend of the F-Re relationship, however, was similar.

Figures 1 and 2 show the data with dilute polymer solutions. It is clearly seen that although there is some scatter in the data, for low polymer concentrations the mixing times are of the same order as in water, whereas at higher concentrations (notably 250 p.p.m. PAA), the mixing times could be substantially higher.

The data obtained with fiber suspensions are particularly remarkable. In Figures 3 and 4 we have shown the data in comparison to those obtained in water alone. The fibrous additives alone appear to increase the mixing times by as much as 150%. Fiber additives in combination with polymers increase them more significantly. The engineering implication of this observation is an obvious one in that the increased mixing times might offset the reduced power consumption in processes where nonhomegenities are of crucial importance. The interpretation of the phenomena can follow along one of the following lines.

Macroscale interpretation

The locus of the turbulent energy dissipation in a stirred vessel is mainly in the trailing vortex system behind the blades of the rotating impeller. Our understanding of the kinematics and the role of this vortex system has been much improved by the studies of Biesecker (1972), Van't Riet et al. (1973), and Bruijn et al. (1974), and the recent work of Van't Riet and Smith (1975) is particularly interesting since it quantifies the velocity distribution in the vortex system. The study of the work of the last authors indicates that the vortex velocity field has the characteristics of both a free and a forced vortex. Further, it can be concluded from the work by Gadd (1965) and Gordon and Balkrishnan (1972) that polymer solutions show extreme resistance to vortex stretching and that vortex suppression occurs under a variety of conditions. Gyr (1968) has shown that solid particle additives can also cause a vortex suppression. It is thus conceivable that in dilute polymer solutions, the trailing vortex system produced is likely to be somewhat weaker in relation to that in a Newtonian solvent. Now, from the work of Van't Riet and Smith (1975) it appears that the vortexing system has an

extremely high shear field. Indeed, the shear rate in the vortex was found to be around ten times higher than the average shear rate in the vessel. When the distributed tracer solution enters this region, such high shear fields reduce the scale of segregation greatly and thus facilitate the subsequent process of mixing on a finer scale occurring over the rest of the vessel volume. It is conceivable in view of the foregoing discussion that in dilute polymer solutions and fiber suspensions the resistance to vortexing will influence the mixing times adversely. There is some indirect evidence to this conjecture. Ranade and Ulbrecht (1977) have studied the process of gas dispersion in agitated vessels containing viscoelastic polymer solutions and noted larger bubble sizes in comparison to Newtonian solutions of comparable viscosity. The high shear field in the trailing vortex is known to be responsible for creating the dispersion (see Biesecker, 1972; Van't Riet et al., 1973). The reduced extent of dispersing probably implies a weaker shear field in viscoelastic polymer solutions.

Microscale interpretation

The model of the turbulent mixing of miscible fluids used in this work is essentially that described by Brodkey (1975). In this model the action of turbulence (and consequent energy dissipation) is principally taken to be to increase the surface between two eddies. The action of the turbulent velocity fluctuations is to stretch the eddies. The assumption of uniform fluctuating velocity components within the eddy limits such models to high wave number or small scale dissipating eddies. The time scales of such eddies can be estimated by using Kolmogoroff's principle of local similarity. The rates of extension of such surface elements can be thus approximated by $(\epsilon/\nu)^{1/2}$. The order of magnitude of stretch rates involved in our case is of the order of $10^3 s^{-1}$. It is well known that dilute polymeric solutions exhibit extremely high resistance to extensional deformations. This is typified by their high extensional viscosities (Metzner and Metzner, 1970). Typically, a solution of 100 p.p.m. of polyacrylamide may show an extensional viscosity which is about three orders of magnitude higher than the Trouton viscosity of the Newtonian solvent. It is thus evident that in dilute polymer solutions, the surface extension processes which are important for the reduction of nonhomogeneities will be inhibited significantly. This, of course, means larger mixing times which are consistent with our experimental observations.

Alternatively, one can follow the arguments of Patterson (1975) who concluded that the mixing time which is obviously proportional to time constant of mixing defined by Corrsin (1957) as

$$\tau \sim \frac{\lambda^2}{6\nu} \tag{2}$$

will be inversely proportional to the third root of power consumption so that the reduction of the mixing efficiency proceeds at a much slower rate than the reduction of power. This implies that with a maximum torque suppression of about 60% observed in our work, the mixing time will go up by only about 17%. This is in a good agreement with the work of Taylor and Middleman (1974) who studied the turbulent dispersion of a scalar contaminant in a tube flow of aqueous polyethylene oxide. They found that the Taylor microscale λ was increased, and this in turn accounts for the increase of the mixing time constant τ.

Critical Speeds for Complete Suspension in Drag Reducing Additive Solutions

About 115 data points were obtained to examine the critical speeds at which complete suspension could be obtained in drag reducing additive solutions. Combinations

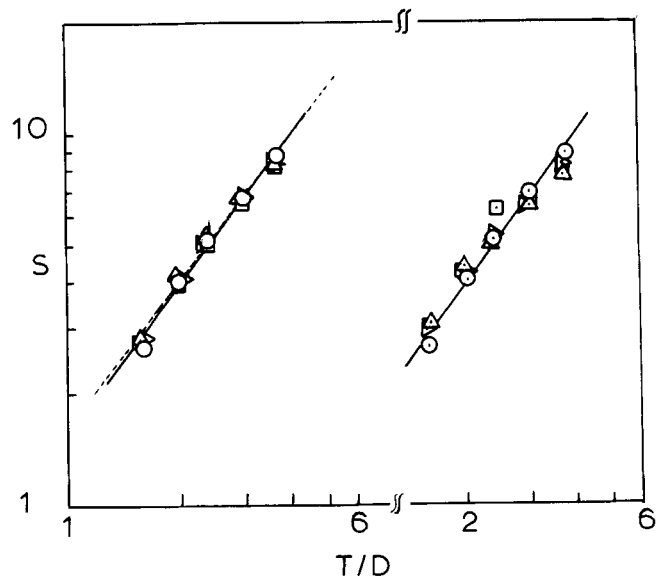

Fig. 5. Influence of DRA on dimensionless critical RPM for suspending 1 mm diam. glass beads in an agitated vessel with a turbine agitator. (Vessel diameter 0.30 m, stirrer diameter 0.10 m). ○ water, △ 50 p.p.m. PAA, > 100 p.p.m. PAA, □ 250 p.p.m. PAA, ◬ 50 p.p.m. PEO, > 100 p.p.m. PEO, □ 250 p.p.m. PEO, ---- Zweitering line.

of bead sizes, agitator, and tank sizes and solutions were used. It is difficult to treat these data individually, and for the purpose of comparison it was thought desirable to use a group devised by Zwietering (1958). He used dimensional analysis to determine suitable dimensionless groups governing the process of suspension for analyzing his experimental data. Expansion of the grouping including the experimentally determined exponents showed that the critical suspension speed n_s is related to the other system parameters by

$$n_s = \frac{s\,\nu^{0.1} x^{0.2} \left(g\frac{\Delta\rho}{\rho}\right)^{0.45} B^{0.13}}{D^{0.85}} \tag{3}$$

where s is the dimensional speed alternatively given by the expression

$$s = \frac{Re_s^{0.1} Fr_s^{0.45} (D/X)^{0.2}}{B^{0.13}} \tag{4}$$

Although data with a variety of agitators were obtained, only the data with turbine agitators will be discussed here, since the trends and conclusions are similar in all the cases. Figures 5 and 6 show the data obtained with 1 and 2.5 mm diam. glass beads using 50, 100, and 250 p.p.m. polyethylene-oxide solutions (WSR 301) and polyacrylamide (AP30), respectively. With the three turbine sizes and two vessel sizes, five multiple points were obtained through which the lines are drawn. One impeller clearance ratio ($h/H = 1/3$) was used throughout this work, since the main objective was to examine the influence of drag reducing additives on the process of suspension. Figure 5 also shows the correlation line obtained by Zwietering (1958). This has been drawn as a dotted line. It is evident that not only is there a good agreement between our experimental data and Zwietering's correlation, but also there is little difference between the water and the polymer solution data.

Figures 7 and 8 show the data obtained with fiber suspensions and their combination with polymer solutions. Note that we treated the fiber suspension as a continuum although the bead sizes were of the same order of

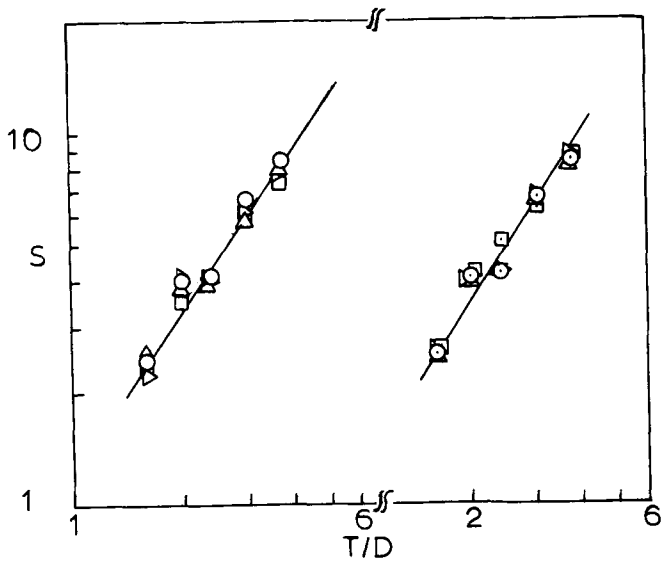

Fig. 6. Influence of DRA on dimensionless critical RPM for suspending 2.5 mm diam. glass beads in an agitated vessel with a turbine agitator. Vessel diameter 0.30 m, stirrer diameter 0.10 m). ○ water, △ 50 p.p.m. PAA, > 100 p.p.m. PAA, □ 250 p.p.m. PAA, ◬ 50 p.p.m. PEO, > 100 p.p.m. PEO, □ 250 p.p.m. PEO.

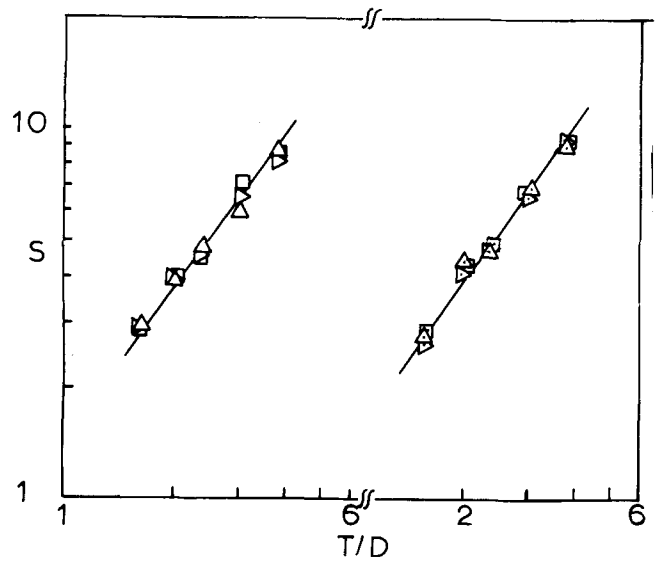

Fig. 8. Influence of DRA on dimensionless critical RPM for suspending 2.5 mm diam. glass beads in an agitated vessel with a turbine agitator. Vessel diameter 0.30 m, stirrer diameter 0.10 m). ○ water, ◬ 50 p.p.m. PAA with 2 500 p.p.m. of fibers, > 100 p.p.m. PAA with 2 500 p.p.m. of fibers, □ 250 p.p.m. PAA with 2 500 p.p.m. of fibers, △ 50 p.p.m. PEO with 2 500 p.p.m. of fibers, > 100 p.p.m. PEO with 2 500 p.p.m. of fibers, □ 250 p.p.m. PEO with 2 500 p.p.m. of fibers.

magnitude as for the fibers. This may be somewhat unsatisfactory, but the agreement with the Zwietering line was again very good. The data do indicate that the suspension process is unaffected by the drag reducing additives.

The interpretation of the solids suspension data will be based again on the same model of turbulence used to interpret the dispersion of a solute.

When a liquid is agitated by a mixer under turbulent conditions, a variety of eddy sizes are generated. The largest velocity scale of the turbulent eddies is of the order of the maximum tip velocity, whereas the largest length scale of the eddies is of the order of the dimension of the agitator. Large scale eddies entrain the particles

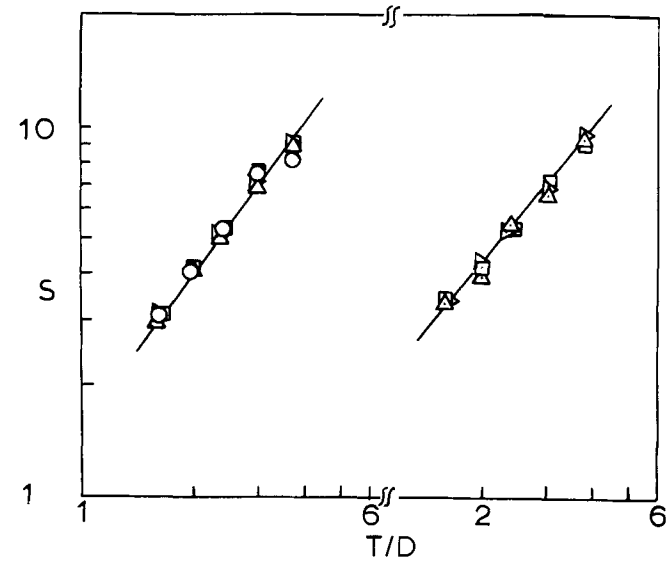

Fig. 7. Influence of DRA on dimensionless critical RPM for suspending 1 mm diam. glass beads in an agitated vessel with a turbine agitator. (Vessel diameter 0.30 m, stirrer diameter 0.10 m). ○ water, ◬ 50 p.p.m. PAA with 2 500 p.p.m. of fibers, > 100 p.p.m. PAA with 2 500 p.p.m. of fibers, □ 250 p.p.m. PAA with 2 500 p.p.m. of fibers, △ 50 p.p.m. PEO with 2 500 p.p.m. of fibers, > 100 p.p.m. PEO with 2 500 p.p.m. of fibers, □ 250 p.p.m. PEO with 2 500 p.p.m. of fibers.

together with portions of fluid adhering to it and transfer both as a single unit. The detailed mechanism of particle suspension in agitated vessels is described by Levich (1962). It appears that when there is a density difference between the fluid and the particle, the entrainment of particles of different sizes needs to be analyzed separately. For particles which are greater than the turbulence microscale, it is evident that the small scale eddies are quite ineffective in keeping the particles in suspension, and one must examine the large scale motions. Using Kolmogoroff's principle of local similarity or, alternatively, using the data provided by Nagata (1975), the scale of such eddies may be estimated to be in the range of about 100 to 1000 μ. On the other hand, the particles suspended in our work were of a dimension of about 1 to 2.5 mm diam. This means that the larger scale motions are responsible for fluid entrainment. In fact, it is conceivable that the solid particles at the bottom of the agitated vessel will be continuously swept by the large scale eddies, and when the velocity scales in these sweeping eddies become larger than the free settling velocity of the particle, the particles are entrained. Since the critical speed for keeping the solid particles in suspension does not change with water and solutions containing DRA, it is conceivable that these large scale motions are unaffected by DRA. Transport processes in the agitated tank which are governed by the large scale motion will thus remain largely unaffected by the addition of DRA.

Practical implications of the findings of this work

In hydrometallurgy, extraction from extremely fine ground particles in a dissolving medium is often important, and this is carried out in agitated vessels. Extraction of gold from the ore by means of cyanide solutions is a typical example. It is quite important to keep the solids in suspension. We have already demonstrated that at least in a certain range, the critical RPM at which all the solids are suspended remains practically the same, whereas DRA addition reduces the energy consumption. It thus appears that considerable power savings should be achievable in such operations. The important problem of mass transfer to or from solid particles in agitated vessels containing

DRA is being studied currently in our laboratory. It is known, however, that the mass transfer coefficient is fairly sensitive to the power consumption below the critical RPM at which the solids are fully suspended, whereas above this point it becomes practically independent of the power consumption (see Kneule, 1956). It is conceivable that the advantages of reduced power consumption may be conveniently combined with the unaltered transport rates. Preliminary data on solid dissolution obtained by Desai (1975) support such a conjecture.

In certain industrial applications, it is important to maintain good circulation patterns in the agitated vessel and to combine these with low turbulence levels. Flocculators (see Lyons, 1967) are typical of this case, where it is important to preserve the floc. It is understandable that the root mean square fluctuations cause intense shear in the neighborhood of the floc particles, and DRA help in reducing this as is evident from the indirect evidence provided in this work. Addition of DRA is likely to be helpful in this regard.

In certain biological reactors, it may be important to handle solid suspensions and yet maintain low turbulence levels in order not to damage the microorganisms. It does appear that polymer addition is frequently done in such cases or polymers such as polysaccharides are generated in situ due to microbial action (Wilson, 1973). The processes here are invariably the ones where turbulent conditions are maintained in the agitated vessels, and rapid circulations are important for avoiding deficiency in oxygen supply. The present work indicates that such processes may not be seriously impeded.

NOTATION

B = weight concentration of suspended solids, %
D = impeller diameter, m
F = dimensionless mixing time, Equation (1)
Fr = Froude number $= DN^2/g$
Fr_s = modified Froude number $= DN^2\rho/g\Delta\rho$
g = acceleration due to gravity, ms^{-2}
h = impeller clearance, m
H = liquid depth, m
N = impeller speed, s^{-1}
N_s = minimum impeller speed to maintain the particles suspended, s^{-1}
P = power input, $Kg\,m^{-2}\,s^{-2}$
Re = Reynolds number $= D^2N\rho/\mu$
Re_s = modified Reynolds number $= D^2N\rho/\mu$
s = dimensionless impeller speed, Equation (6)
T = tank diameter, m
T_L = Lagrangian integral time scale, s
X = particle size of the solid, m

Greek Letters

θ = mixing time, s
λ = Taylor microscale, s
μ = viscosity of the liquid phase, Ns/m^2
ν = kinematic viscosity of liquid, m^2/s
ϵ = specific rate of turbulent energy dissipation, J/Kg^s
ρ = liquid density, Kg/m^3
ρ_s = density of the solid particles, Kg/m^3
$\Delta\rho$ = $\rho_s - \rho$
τ = time constant for decay of concentration fluctuations of a scalar contaminant, s
ϕ = constant, Equation (3)

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